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## 2. The Effective Teacher of Dyscalculic Children

Our research suggests that the most effective teachers of children with dyscalculia share a range of key characteristics as teachers.

Many of these characteristics are explored within this volume. But because it is the issue on which we are questioned more than any other, here is a brief summary of some of the more important features that effective teachers generally show.

### *Generally speaking effective teachers of children with dyscalculia:*

- Always move from instruction to restricted practice, in which it is hard for the pupil to make an error.
- Ask lots of specific questions to explore learning.
- Constantly change the methods of teaching to suit the situation.
- Correct all errors and give hints rather than straight answers.
- Encourage shared learning.
- Ensure that each small point has been mastered, rather than ploughing on through a major topic.
- Explain in small steps, one point at a time.
- Give lots of feedback which is always positive.
- Practise what has been learned with the pupils.
- Provide lots of examples.
- Provide rules and define points, coming back to these rules and points as the lesson progresses.
- Revise points as they go.
- Spend much more time than non-effective teachers demonstrating points.
- Spend much more time than non-effective teachers explaining points to pupils.
- Stick to the point and avoid wandering, either because of their own asides or through pupil interruption.
- Support pupils when introducing new concepts which may be confusing, allowing the pupils to discuss matters with each other as much as possible.
- Use visuals and key words as reminders of the whole skill or topic that has been taught.
- Work at a high speed, keeping a high energy level in each lesson at all times.
- Work in a multi-sensory approach – at least providing visuals alongside the explanation.
- Work in sequences, and ensure that the pupils know where the sequence starts and ends.
- Work with all pupils by arranging it that everyone can respond – at least in writing if not verbally. (Shared learning and group work obviously help).

### *Generally speaking effective teachers with dyscalculic children tend not to:*

- Ask pupils if they have understood (they test specifically for understanding).
- Ask the whole class if there are any questions (there might be, but the majority will be lost at this point). Errors in understanding are picked up through the testing and checking which is part of the complete lesson.
- Ask non-academic questions.
- Merely correct wrong answers from students – they move into explorations of the error.

It is these various factors that we have integrated in this book, working on the effective teaching methods that appear to benefit virtually every child with dyscalculia, and creating this approach to teaching and learning.

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### 3. Methods of Teaching

#### 3.1. Can teaching of dyscalculic pupils be made more efficient?

We start from the premise that almost all children with dyscalculia can learn maths if the methods of teaching are changed to suit their particular needs?

But this raises an important point. There is no point reading about methods of teaching unless you believe it is possible to improve the efficiency of your teaching. That is to say, discussions of methods of teaching are about getting more teaching and learning out of the same teacher, the same resources and the same pupils, by changing the methods of teaching utilised.

The issue is rather like learning to swim. It is quite possible to learn to swim, and avoid drowning, without being much good at swimming. Someone who has studied swimming technique however might be able to show you how to swim in a much more efficient way – more output for less input. More result for less stress. Even an expert swimmer can improve – can squeeze that extra one hundredth of a second out of swimming a length. The same is true in teaching.

You might be having considerable success with some dyscalculic pupils in your school. What we suggest is that you could have even more success, if you changed some of the methods of teaching you use. But we know that if our suggestions involve your taking up many more teacher-hours with these pupils, you are likely to object that the time is not available. So, if the aim is to be more effective, without taking more time, you have to be more efficient. However, increasing efficiency is not a simple proposition.

The complexity of improving output was revealed in the famous Jastrow case in 1900 involving the introduction of the Hollerith tabulating machine into the US Census Bureau.

The inventor of what was in effect the first desk top computer suggested that the operators should enter about 500 records a day and this they duly did. If they ever exceeded this level of work, this caused high levels of stress among the workers. Worried by the stress levels the government insisted that the next batch of trainees should not be told how many cards they could process - and within two weeks of training this new group was processing over 2000 cards a day without any stress or other ill effects.

The point is that although we may feel as if we are doing everything we can, and feel that any extra work would be quite impossible, these feelings may not be a valid measurement of reality. It may be that we could each of us teach more effectively with less effort.

This view is supported by the realisation that although most teachers learn about alternative methods of teaching maths on INSET courses, some still stick to their own favoured methods. This is not because these methods produce better results but because these teachers are happier with their tried and trusted methods.

Efficiency is not a word often associated with schooling - but there really is no reason why it should not be so linked. Four classic texts on school improvement topics in general (Harris 1997, Bennett 1995, Fullan 1991 and Hopkins 1994) muster only a single index entry on the subject for the 1000+ pages of text between them. The education library at the University of Nottingham, one of the major international centres for the study of school improvement, contains five efficiency references - all regarding the installation of efficient heating systems in schools.

Because efficiency is so rarely mentioned in education we must ask, is there some reason why this concept might be invalid in the educational context? Just because those in business and industry talk in this way, it does not mean that it is an inappropriate notion for education.

In terms of economics and democracy, most of us would surely agree that the search for efficiency is as natural in education as anywhere else. Of the approximately £18 billion a year spent on education in the UK, 93% comes from public funds - our money as taxpayers. Even if it did not, and a suddenly generous Bill Gates funded education, we should surely still seek to be more efficient in schools, simply out of a professional desire to do our jobs to the best of our ability.

Thus on the face of it efficiency ought to be a valid concept in education. Let us try to give an example of how it might work in a school.

The head of maths might note that it currently takes ten hours to teach the principle of multiplication to a group of children who have particular difficulties with maths. The desired outcome, where every pupil gains 90% or above on a test at the end of the ten hours, is achieved in 95% of cases. Why does the teacher not ask whether, with a different approach, these same pupils could reach the same level of understanding in five hours? If so, the speed at which these possibly dyscalculic pupils catch up with their more mathematically able colleagues could be doubled.

Opponents in the school might argue that the “five hour” programme was too intense, that the “ten hour” pupils had more fun, or learned to love the subject more. The “five hour” programme (it might be argued) gave the same knowledge but the pupils hated the work. This is an unlikely scenario but if true, we would need to redefine our aims. We should seek the most efficient way of teaching the topic so that the pupils not only gain a certain level of knowledge and the ability to use it in certain conditions, but could also express a certain measure of joy in the lessons (as measured on our patented joyometer). We would then seek the most efficient way of delivering the whole package. If we could do it in seven hours and retain the joy, then that teaching programme is more efficient than the ten hour programme, and we would adopt it.

If we could make all teaching just ten percent more efficient (i.e. achieving the same results using only nine hours instead of ten), it would be equivalent to adding another maths or special needs teacher to every secondary school in the UK. This is equivalent to increasing the UK school budget by over £300 million per annum!

Of course I might ask, what am I going to do with my spare hours? Possible answers include teaching in greater depth to ensure all pupils become competent at maths, all dyscalculia is overcome, and so on. It seems a fairly good objective.

How do we actually increase classroom efficiency? Clearly we should not lose sight of the fact that we are working with people. The school does not become a machine. Nevertheless, it is inescapable that both effective teachers and effective departments (both maths and special needs) are more efficient in the process of teaching and learning. Because they do it better, teachers gain more satisfaction for a job well done, and at this point stress levels decline. Through this process, the learning experiences of the pupils are expanded and, as we all know, a wide range of experiences is itself beneficial from the teaching and learning point of view.

We therefore now turn to various high efficiency methods of teaching maths to dyscalculic pupils.

### 3.2. Making maths teaching multi-sensory

Where possible the pupils' maths experiences must be multi-sensory. Multi-sensory work was introduced by Montessori, and later taken up by special needs teachers who sought to find ways of improving the teaching of literacy skills to dyslexic pupils. Bryant and Bradley at Oxford University later proved that the multi-sensory method is the most efficient method of teaching for all pupils, irrespective of whether they are dyslexic or not. Virtually every book on dyscalculia agrees that multi-sensory methodology works best with pupils who have particular problems in maths.

From its origins multi-sensory teaching recognised that the traditional approach, which involves the teacher explaining and the pupil listening, is unlikely to be very effective as the pupil is only using one sense. Instead the pupil should be involved in work which involves at least two senses simultaneously, or in close proximity. This is true for all pupils. It is doubly true for dyscalculic pupils.

This can be achieved by such activities as:

- The pupil saying and reading problems simultaneously, and discussing and writing solutions one after the other.
- The pupil making graphical representations of problems with models, diagrams, cards, or counters while saying or reading the problem.
- The pupil hearing numbers while seeing or touching counters. The pupil picking wooden letters out of a bag in the right order, in order to add a sense of touch to spellings. The pupil entering or locating a date on a time line each time a date is mentioned.
- The pupil imagining smells, sounds and visual images "attached" to the problem being studied.
- The pupil drawing a spider diagram to explain all elements of the topic being learned.
- The pupil working with a colleague to solve a problem, each talking the problem through and writing the solution down as they go (see the section on shared learning, below).

Examples of multi-sensory work appear in part five of this volume.

### 3.3. Structured instruction

If we wish to be more efficient we need to follow the view of John Carroll (1971) whose view that virtually anyone could learn virtually anything is one we have adopted.

Carroll focused on two important concepts:

- Different pupils need to be taught in different ways.
- Different pupils take different amounts of time to get to a set standard.

Never is this truer than when working with dyscalculic pupils. The implication of this approach is that as teachers we should amass a range of materials and teaching techniques and adopt the most appropriate for each topic or concept, and for each pupil's individual needs. Our work will be made much easier if:

- Each topic is broken down into individual units of approximately single lesson size.
- Pupils can see success as each small unit is completed.
- Pupils learn self-initiation and self-direction in learning.
- Problem solving is utilised as soon as possible in each teaching programme.
- Pupils are motivated to succeed.
- Progress is evaluated very frequently to make self-direction more effective.

To make this approach work we need a set of objectives which tell the pupils what they are learning and what they ought to be able to do by the end of each session. As Bruce Joyce (1996) points out, objectives must be stated using words such as:

- Solve (Given any two numbers under 500 pupils can add or subtract correctly where the answer is positive).
- State (Pupils can state the number of sides in a triangle and square).
- Explain (Pupils can explain the meaning of "odd numbers").
- List (Pupils can list all the numbers between 1 and 100 which are exactly divisible by 9).
- Describe (Pupils can describe an isosceles triangle).

On the other hand objectives should not be stated using words like "understand", "appreciate", "know" and "comprehend" as they are difficult to examine, and thus we never know if learning is effective or not.

Thus we argue that...

#### ***Before we do anything else we need to do two things:***

- Divide the basic topics which the pupil has not grasped into a range of key areas if this has not already been done so. There should ideally be no more than a dozen or so headings. This special syllabus must be structured so that when the pupil comes to a later part of the course she or he already has all the knowledge that will be needed to learn this new concept.
- Find out what the pupil knows in each area of the mini-syllabus. We thus need a set of tests which exactly match the structured sequence in which the course is taught. For example, if one is teaching pupils about fractions it is important to be sure that the pupil grasps the basic concept of division, what a fraction is and so on. If one wishes to teach the division of fractions one cannot proceed without the pupil knowing and being able to use basic multiplication tables. If the pupil is still likely to get  $6 \times 7$  wrong, then there is no point in teaching division of fractions.

Where pupils are not permitted to progress to later activities until they prove their knowledge of earlier learning the speed of learning may initially seem to be slower, but in reality it increases radically because all subsequent learning is securely based on earlier learning. Overall learning during the course of a year should increase by at least 25% if this approach is used without any exceptions.

I initially introduced this technique in the book *Maths and Dyslexia* after finding that large numbers of pupils supposedly suffering from dyscalculia could be helped greatly if we analysed exactly which preliminary concepts they were misunderstanding.

The approach has now been refined in *Tests for Dyscalculia* (First and Best 2002).

### 3.4. The rules of structured instruction

We believe that each and every lesson should proceed through the following 20 steps, in this specific order:

1. Actively review (with pupil participation) selected items from past lessons that are relevant to the topic of the day. Thus you might have ended the last session with a clear view that the pupil had mastered the concept of adding two numbers each of between 10 and 90, where the answer is anything between 20 and 99.
2. Actively review the last lesson, reminding the child that he or she really does now have this concept understood. Give praise for this achievement, and remind the pupil how far she or he has come.
3. Establish with the pupils a lesson structure which orientates the pupils. This gives the pupils a road map – a sense of direction. This is not just another lesson – it is clearly the lesson in which you now tackle addition where the answer can go over 100.
4. Discuss the objectives and targets for the lesson with the pupils. This is part of making the pupils your partners in the planning and structuring of the work. Make the target overt and testable.
5. Provide clear instructions – all the research suggests that the most effective way of helping dyscalculic pupils is by spending much more time at this stage. If this section is shortened, then the child can have real problems subsequently.
6. Show the pupils the materials to be used.

*This preliminary set of activities leads directly into the heart of the lesson:*

7. Present new material in small steps checking that the pupils grasp each step. Never move more than one step at a time. So if you are teaching:

$$\begin{array}{r} 44 \\ + \underline{86} \end{array}$$

it is important to ensure that  $4 + 6$  is grasped, and that an effective way of handling this concept is used by the pupil.

What's more, if the child is habitually using a different technique (such as adding the tens before the units) this will become apparent here. You will then have the option of deciding whether to proceed to show the child that this is a process very likely to give wrong answers, or if you have a case in which the child is happier working in this way and is not making errors. Our strong view is that there is no point in changing a child's method of working – no matter how odd, eccentric or downright bizarre it appears to be, unless it is a method that fails to work. Just because a child appears to work backwards, it does not mean the child is wrong.

8. Work in a multi-sensory way at each stage, letting pupils handle physical representations of the process, with opportunities to say the key words and write down details as they proceed. "84" needs to be said, written, set out in counters (8 large "tens" and 4 small units, or whatever is appropriate), and overall conceived as something real and meaningful.

9. Give many varied examples as you work, changing the senses used. The pupils must read, hear, say, see and touch examples of the work in progress.
10. Give narrative demonstrations of the concepts in question while pupils manipulate physical examples.
11. Stay on subject: don't get side-tracked by such issues as faulty equipment, late arrivals, discipline issues, etc. Save the joke which helps the pupil-teacher rapport for the end of the lesson. The lesson should be so fast moving that there is no time for interruption. This is true in most lessons – although particularly skilled teachers can often break this rule without the lesson suffering. But when dealing with dyscalculic pupils for whom the whole concept of maths is alien, we think it is strongly worth bearing in mind.
12. Review the rule or topic that has been taught.
13. Discuss the topic with the pupils, check understandings, ask for demonstrations of understanding from pupils, and return to point nine if there are misunderstandings. In such a case give very high levels of immediate feedback so that if a pupil is making an error he/she knows about the error and the reasoning immediately. This approach should be corrective and respectful.
14. Test the knowledge of all students so that pupils can experience the success of having learned the topic.

***We now move on to practising the point that has been taught:***

15. Move the pupils towards mastery of the topic through repeated practice of each step in the procedure, using high levels of corrective feedback.
16. Where any errors still occur, or where earlier errors return, retrace each step in the procedure, removing errors in a consistent manner.
17. The pupils now undertake independent practice, solving problems and using shared learning techniques (see below).
18. The topic is not concluded until there is over 90%-95% accuracy.
19. The pupils are congratulated on reaching yet another target.
20. Practice continues over time, coming back to make sure the ideas don't slip away from the pupils. The practice becomes the start of the next lesson on the next topic.

Following the 20 steps over and over again enhances learning outcomes dramatically always providing that the point being taught is simple, clear and just one step on from the last point mastered.